

LESSON 2

Measuring Income Inequality: The Lorenz Curve and Gini Coefficient

INTRODUCTION

In order to have a meaningful discussion around income inequality one has to understand how it is measured. The most commonly discussed measures by economists and politicians are the Lorenz Curve and Gini Coefficient. In this lesson, students construct a Lorenz Curve, learn how it is related to the Gini Coefficient and look at data for Canada and other nations on these measures. This lesson will allow students to engage in higher-level discussions around income inequality, and differentiate it from poverty, once they have a firm grasp of how they are measured.

Backgrounder

The two most common measures of a country's income inequality are The Lorenz Curve (a visual indicator) and the Gini Coefficient (a mathematical indicator).

The Lorenz Curve was developed by American economist **Max O. Lorenz** in 1905. A Lorenz curve is drawn by plotting a country's cumulative income and cumulative workforce. The Perfect Equality Line (drawn at 45 degrees) shows a country in which income is perfectly equally distributed, each 10% of the population receives 10% of income. The closer a country's Lorenz curve is to the line of perfect equality, the more equally distributed is their income. A larger gap between the perfect equality line and the Lorenz Curve indicates greater inequality.

The Gini Coefficient was developed by the Italian statistician and sociologist **Corrado Gini** (and published in his 1912 paper Variability and Mutability)—it is the most commonly cited measurement of inequality. It provides a single number what the Lorenz Curve illustrates visually giving it the advantage of being relatively easy to understand and brief. For those who love math, the Gini coefficient is the ratio of the area between the line of perfect equality and the observed Lorenz curve to the area between the line of perfect equality and the line of perfect inequality. The gini ratio is always between zero and one, where zero equals perfect equality and one equals perfect inequality (of course most countries fall in between).

For an easy way to teach the Lorenz Curve and Gini Coefficient in your class, watch this You Tube video: <https://www.youtube.com/watch?v=ns9f2FOc26M>

For a more detailed treatment of these concepts please refer to **Appendix A**.

ECONOMIC CONCEPTS

- Gini Coefficient
- Income Inequality
- Lorenz Curve

TIME

1 hour

LEARNING OBJECTIVES

Students will be able to:

- Create and interpret a Lorenz Curve illustrating distribution of income.
- Interpret a Gini Coefficient representing distribution of income.
- Understand how Canada ranks in comparison to other nations in income inequality using the Gini Coefficient statistic.

VISUALS

- Visual 2.1 - Extreme Income Inequality in South Africa
- Visual 2.2 - Building a Lorenz Curve: The Citizens of Maple Land
- Visual 2.3 - Gini Coefficients by Country

HANDOUTS

- Handout 2-1
- Handout 2-2 (student's)
- Handout 2-2 (answer guide)

MATERIALS

- 1 copy per student of Handouts 2-1 and 2-2

TEACHER'S GUIDE: PROCEDURE

1. Explain to students that income inequality exists in every country and is not necessarily good or bad (other lessons will investigate these kinds of normative questions while this lesson is strictly a positive analysis around calculating income inequality).
2. However sometimes economists and politicians become concerned about income inequality as it becomes more pronounced. In other words, when many individuals in a population earn a very small share of the income and most income goes to a few wealthy individuals. Show the students the pictures in **Visual 2-1** and explain that South Africa is an example of a country with a high degree of **income inequality** based upon the usual measures that economists and policy makers use.
3. Explain that most governments, including Canada's, make some attempt to redistribute income to help the least fortunate – meaning they take tax dollars from high-earning individuals to provide money and services to low-income earners. This act of redistribution lowers the level of income inequality in a society.
4. In order to make these decisions, it is important first to measure the income distribution in a nation. In this lesson, students will learn two such measures: the **Lorenz Curve** and the **Gini Coefficient**.

ACTIVITY – BUILDING A LORENZ CURVE FOR THE CITIZENS OF MAPLE LAND

5. Explain to students that the first step in graphing a Lorenz Curve is collecting income data for a population. Provide each student with a copy of **Handout 2-1**. Students may work through the example with you on their worksheet while you display the results using **Visual 2-2**. For this example, we will imagine a small country, Maple Land, with just five citizens: Scott, Seth, Bob, Joe and Bonnie. Their income data is provided in Handout 2-1.
6. Note that the citizens of Maple Land do not have equal income. Ask students to add up the total income for these five residents (\$500). Ask students: If there were equal distribution of income, how much would each citizen of Maple Land earn? (\$100)
7. Ask students to sort the individuals and put them in the table on Handout 2-1 in ascending order. In other words, Seth – who is earning the least at \$15 – goes in the top row of the table. Show the step 1 of Visual 2-2 so students can check their work.

8. Explain that next students need to calculate each individual's share of the total income. Ask students: How will you calculate Seth's share of the total income? ($\$15/\$500 = 3\%$) Ask students to repeat this procedure for each individual and fill in the third column in Handout 2-1. Again, students can check their work using step 2 of Visual 2-2. Explain that the final column on the table represents the cumulative share of income going to 1 worker or 2 workers or 5 workers.
9. The lowest paid worker, Seth, receives 3% of the income. The lowest paid two workers, Seth and Bob, earn 3% + 14% of the income, so cumulatively (together) they earn 17% of the income. All of the workers, together, earn 100% of the income. This might seem strange to students, but explain that this column is important for graphing a Lorenz Curve.
10. Ask students to calculate the cumulative share of the income for 3 and 4 workers and fill in the final column in their worksheets. Once students are done, show step 3 of **Visual 2-2**.
11. Direct students to the graph at the bottom of Handout 2-1. Ask students what they think the black line on the graph at a 45 degree angle represents. (It represents perfectly equal distribution of income, referred to as "the perfect equality line". The lowest 1% of income earners have 1% of the income. The lowest 20% have 20% of the income. The lowest 50% have 50% of the income, etc.) Have the students plot the data they have calculated onto this graph. The x-axis represents the cumulative percent of households (in our simple example with five residents, each individual adds 20%). The y-axis represents the cumulative percent of income. Seth represents 20% of the income earners in Maple Land, and he earns 3% of the income, the first data point should be plotted at (20%, 3%) and so on. Reveal step 4 of Visual 2-2 so students may check their work.
12. Tell the students they have now constructed a Lorenz Curve. The larger the area between their curve and 45-degree perfect equality line signifies larger shares of income inequality. So the further below the 45-degree line the Lorenz Curve is the more income inequality in the society.
13. Tell the students that the actual calculation of the Gini Coefficient is quite complicated mathematically. However, the Gini varies between 0 and 1. A Gini of 0 represents perfect equality and a Gini of 1 perfect inequality. The larger the area between the line of perfect equality (45-degree line) and the Lorenz Curve the larger the Gini Coefficient (and amount of inequality). The larger the Gini, the more inequality.

14. Show **Visual 2-3**. Tell the students that this chart displays recent data on Gini Coefficients for many countries in the world. Ask the students which country on the chart has the largest degree of income inequality (South Africa). Ask the students where Canada ranks on the chart. Does it have more or less income inequality than the United States? (Canada has less income inequality) Mexico? (Canada has less income inequality).
15. Place the students in groups of 2-4 and have them work through **Handout 2-2**. An answer guide is supplied (**Handout 2-2 – Answers**). Once the groups have completed their work, have a class discussion around the correct answers to the questions.

DEBRIEF

Debrief the lesson by asking the students the following questions:

- Briefly describe how a Lorenz Curve is created.
- What is the relationship between a Lorenz Curve and the Gini Coefficient?
- What does it mean if a country has a Gini Coefficient close to 1? Close to 0?
- Name some countries with high levels of income equality. Low levels of income inequality.

CONCLUSION

Income inequality is a hot topic among economists and politicians. Yet many cannot explain how it is measured. This introductory lesson helps students understand how to construct a Lorenz Curve using income data and how that curve is related to the commonly cited Gini Coefficient.

Bonus Activity

Ask the students to use the CIA's *World Factbook* to find the Gini Coefficients for five nations.

<https://www.cia.gov/library/publications/the-world-factbook/rankorder/2172rank.html>

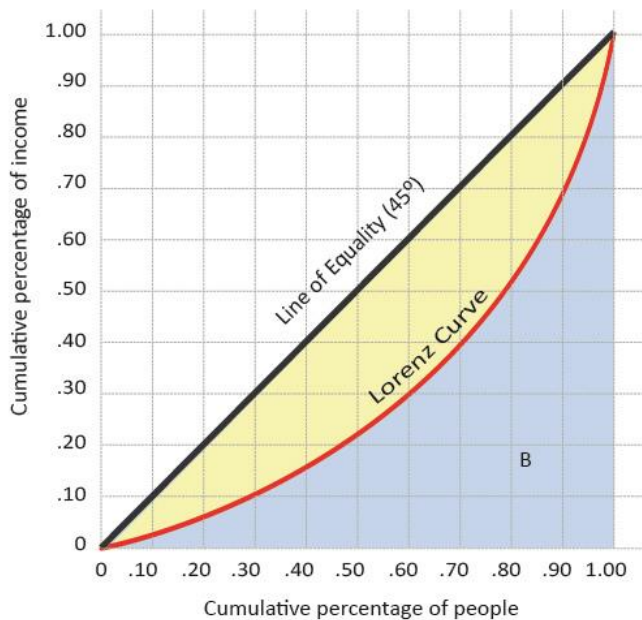
Ask them to discuss any trends they see in the data they collected.

Appendix A: Lorenz Curve and Gini Coefficient

(Source: Towards a Better Understanding of Inequality in Canada, p. 116)

The Gini Coefficient is an index of income equality that has been used for many years and in many countries. Its nature can best be understood by considering the Lorenz Curve (figure A1), which measures along the horizontal axis the cumulative percentage of people with income from the lowest to the highest level and along the vertical axis the cumulative share of income earned by them. The 45° line represents a situation in which income is distributed equally. The two axes represent total inequality as one person earns all income in the country. The curved line shows a realistic degree of equality similar in nature to that found for Canada and most countries in the world.

Figure A1: The Lorenz Curve



The calculation of the Gini Coefficient basically involves dividing the area labeled A by the areas labelled A plus B in figure A1. In other words, the Gini Coefficient (G) is $G = A \div (A + B)$. If the Lorenz Curve coincides with the 45° line, the denominator of the equation is zero so that G is also zero and income is perfectly equal. If the Lorenz Curve coincides with the two axes, B is zero and the ratio G is equal to one, reflecting perfect inequality. The arithmetic formula involved in the calculation of the areas A and B using the basic data on family incomes is complex and need not be discussed here.

VISUAL 2-1: Extreme Income Inequality in South Africa



A \$5 million home in South Africa



Kiptown, South Africa

VISUAL 2-2: Building a Lorenz Curve

The Citizens of Maple Land

The Data

Name	Income
Scott	\$90
Seth	\$15
Bob	\$70
Joe	\$200
Bonnie	\$125

Step 1

Name	Income (ascending)
Seth	\$15
Bob	\$70
Scott	\$90
Bonnie	\$125
Joe	\$200

Step 2

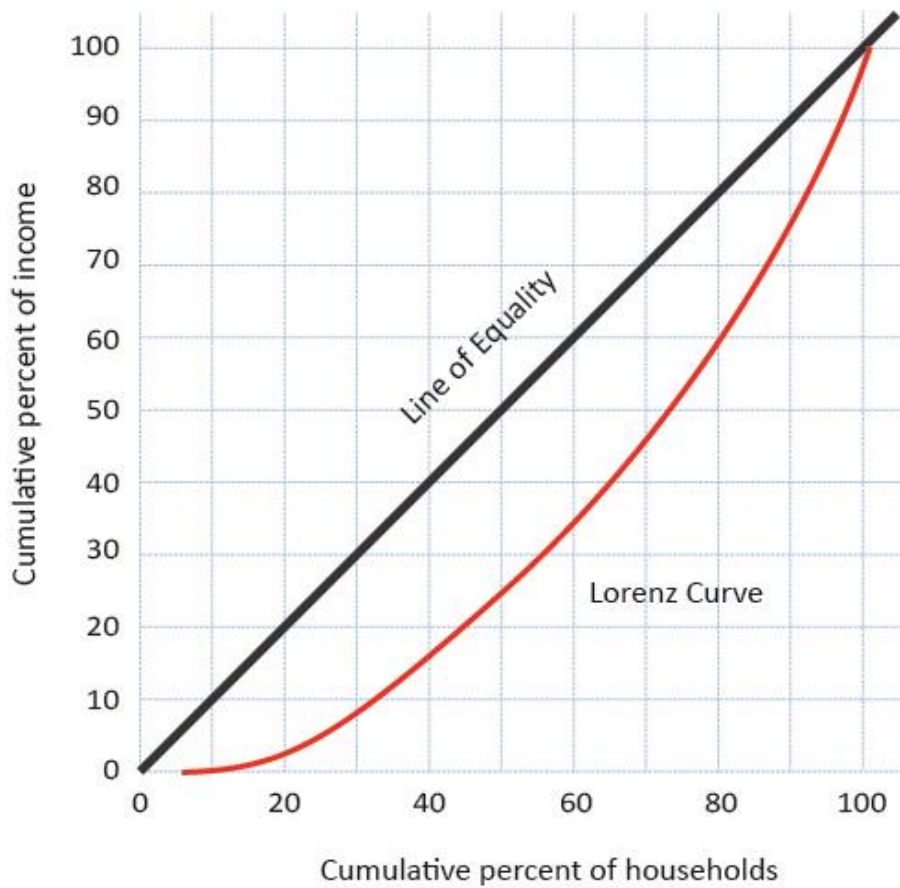
Name	Income	Share of Total Income
Seth	\$15	$\$15/\$500 = 3\%$
Bob	\$70	$\$70/\$500 = 14\%$
Scott	\$90	$\$90/\$500 = 18\%$
Bonnie	\$125	$\$125/\$500 = 25\%$
Joe	\$200	$\$200/\$500 = 40\%$
TOTAL	\$500	$\\$500/\\$500 = 100\%$

Step 3

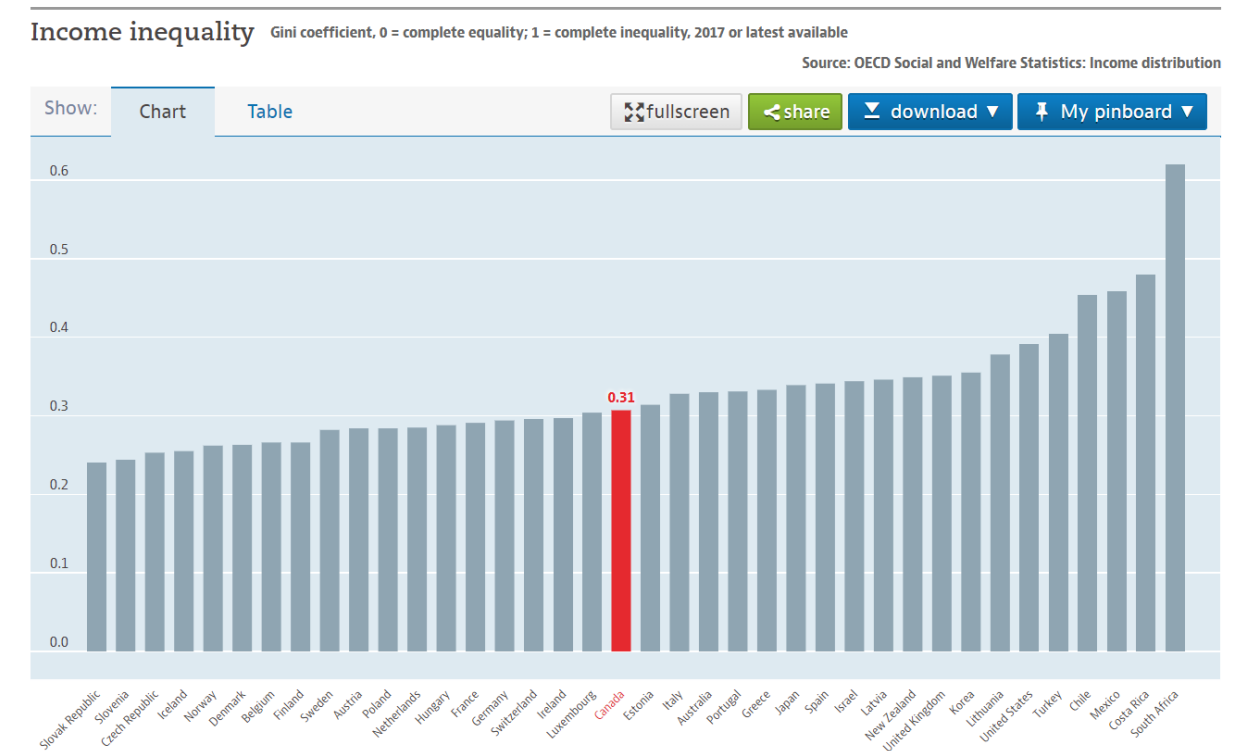
Name	Income	Share of Total Income	Cumulative Share of Total Income
Seth	\$15	$\$15/\$500 = 3\%$	3%
Bob	\$70	$\$70/\$500 = 14\%$	$3\% + 14\% = 17\%$
Scott	\$90	$\$90/\$500 = 18\%$	$17\% + 18\% = 35\%$
Bonnie	\$125	$\$125/\$500 = 25\%$	$35\% + 25\% = 60\%$
Joe	\$200	$\$200/\$500 = 40\%$	$60\% + 40\% = 100\%$
TOTAL	\$500		

Step 4

The Lorenz Curve



VISUAL 2-3: Gini Coefficients by Country



Source: OECD (2019), Income inequality (indicator). doi: 10.1787/459aa7f1-en. (Accessed on 01 July 2019.)

RESOURCES

CIA's World Factbook: <https://www.cia.gov/library/publications/the-world-factbook/rankorder/2172rank.html>

YouTube Video on Measures of Inequality: <https://www.youtube.com/watch?v=ns9f2FOc26M>

Towards a Better Understanding of Income Inequality in Canada, Fraser Institute
<https://www.fraserinstitute.org/sites/default/files/towards-a-better-understanding-of-income-inequality-in-canada.pdf>

